Semester- 2
PHYCC204: Waves and Optics
Unit- 3

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Interference:

When two waves meet under specified conditions then there is sustained destruction or constructive reinforcement of two waves. This phenomenon is called interference.

When effect of one wave is constantly neutralized by the other then such interference is called destructive interference and when these effects reinforced then they are said to be constructive interference.

Light is a wave so there must be interference in light. When light interfere constructively it produces brightness and when it interferes destructively it produces darkness.

Condition for interference:

1. The two waves must be coherent ie, two waves phase difference is independent of time.
2. The two waves must be of same amplitude or nearly same amplitude and frequency
3. They must travel along the same line or along different lines at small inclination and they must be linearly polarized.

For darkness path difference of waves must be an odd integral number of half waves.

Path difference = $\frac{(2s+1)\lambda}{2}$ where s = 0,1,2,……..

For brightness path difference of waves must be any integral multiple of full waves.

Path difference = sλ where s= 1, 2, 3……..

Theory:

Suppose two light waves of same amplitude a and wavelength lambda from two sources S_1 and S_2 move along the same direction and meet at a point p

y_1, y_2 = displacement of the plane waves P at time t

x_1, x_2 = displacement of part of sources and phase difference of the waves

$y_1 = a \sin(\omega t - \frac{2\pi}{\lambda} x_1)$

$y_2 = a \sin(\omega t - \frac{2\pi}{\lambda} x_2)$

Resultant displacement at P is $y = y_1 + y_2$

$y = a \sin(\omega t - \frac{2\pi}{\lambda} x_1) + a \sin(\omega t - \frac{2\pi}{\lambda} x_2 - \delta)$

This is an equation of simple harmonic motion whose amplitude is

$$2a \cos [\frac{\pi}{\lambda} (x_1 - x_2) - \frac{\delta}{2}]$$

Intensity is proportional to square of amplitude.

Therefore, intensity of light at P $= k \cos^2 [\frac{\pi}{\lambda} (x_1 - x_2) - \frac{\delta}{2}]$

Where k is a constant

Here intensity at P will not be constant unless lambda is a constant. Here it is independent of time. This was the first condition for interference ie, waves must be independent of time.

Condition for complete darkness is

$\delta = a$

when $\delta = 0$

$P = k \cos^2 [\frac{\pi}{\lambda}(x_2 - x_1)]$

Intensity at P is zero when

$\cos [\frac{\pi}{\lambda}(x_2 - x_1)] = 0$

$\frac{\pi}{\lambda}(x_2 - x_1) = (2s + 1) \frac{\pi}{2}$

$(x_2 - x_1) = (2s + 1)\frac{\lambda}{2}$

ie, Path difference = $(2s + 1)\frac{\lambda}{2}$

Intensity at P is maximum when

$\cos[\frac{\pi}{\lambda}(x_2 - x_1)] = +1, -1$

$\frac{\pi}{\lambda}(x_2 - x_1) = s\,\pi$

$(x_2 - x_1) = s\,\lambda$

Path difference = $s\,\lambda$

There are two ways of production of coherent waves:

- Division of wavefront
- Division of amplitude

Division of wavefront

Wavefront is the locus of points where the particles vibrate in phase. If a wavefront is divided into two parts or two small parts of it are selected and rest is neglected then we will have two sources having no phase difference. The waves from these sources will also have no phase difference. There will be phase difference only when they traverse different path before meeting to interfere.

Division of wavefront is produced by following experiment:

- Young's experiment of interference
- Fresnel's biprism
- Single mirror experiment

Division of amplitude

In this method wavefront is divided into two wavefronts, one by reflection and other by refraction. Since two wavefronts originate from common wavefront the wave corresponding to the wavefronts are coherent. The amplitude of the wave will be different.

This type of coherent wave front can be produced by

- Newton's ring

- Michelson interferometer
- Fabry Pérot interferometer

<u>Interferometer</u>

It is an apparatus to obtain interference fringes with a path difference be interfering rays.

When it is used to determine refractive index of substance it is called refractometer.

<u>Fabry-Perot Interferometer:</u>

It is based on the principle of multiple beam interferometry.

<u>Multiple Beam Interferometry</u>

Interference pattern which is obtained by many beams which are derived from a single beam by multiple reflection or division of amplitude. When a plane wave falls on a plane parallel glass plate then beam will undergo multiple reflections at the two surfaces and as a result large number of beams of diminishing amplitude will emerge on both sides of the plate. These beams interfere to produce interference pattern at infinity. Fringes thus formed are much sharper than those formed by beam interference.

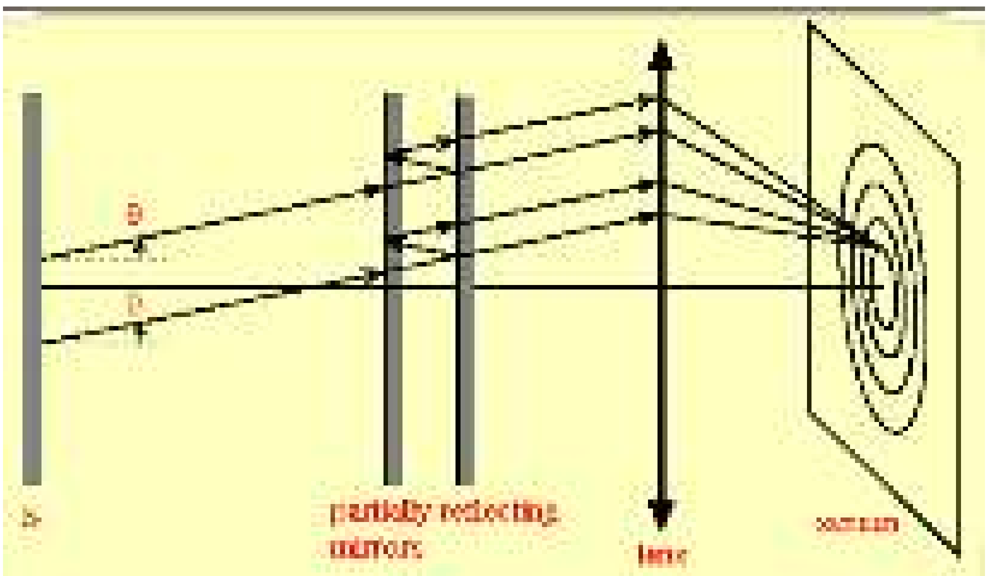
The Fabry-Pérot interferometer (variable-gap interferometer) was produced in 1897 by the French physicists Charles Fabry and Alfred Pérot. It consists of two highly reflective and strictly parallel plates called an etalon. Because of the high reflectivity of the plates of the etalon, the successive multiple reflections of light waves diminish very slowly in intensity and form very narrow, sharp fringes. These may be used to reveal hyperfine structures in line spectra, to evaluate the widths of narrow spectral lines, and to redetermine the length of the standard metre.

- The Fabry-Perot interferometer is a high resolving power instrument, which makes use of the ‘fringes of equal inclination’, produced by the transmitted light after multiple reflections in an air film between two parallel highly reflecting glass plates .

- For high mirror relativities, such a device can have very sharp resonance.

- Based on these sharp features, distance can be measured with a resolution far better than the wavelength.
- The net phase change is zero for two adjacent rays, so the condition 2d $\cos\alpha = m\lambda$ represents an intensity maximum.

It consists of two reflecting mirrors that can be either curved or flat. Only certain wavelengths of light will resonate in the cavity: the light is in resonance with the interferometer if $m(\lambda/2) = L$, where L is the distance between the two mirrors, m is an integer, and λ is the wavelength of the light inside the cavity. When this condition is fulfilled, light at these specific wavelengths will build up inside the cavity and be transmitted out the back end for specific wavelengths. By adjusting the spacing between the two mirrors, the instrument can be scanned over the spectral range of interest.



<u>Intensity Distribution:</u>

T and R = fraction of intensity transmitted and reflected

$\sqrt{T}$ *and* $\sqrt{R}$ = fraction of amplitude transmitted and reflected

a = amplitude of incident wave

a, aT, aTR, aTR^2…..= successive transmitted ray through E_1 and E_2.

Therefore,

Phase difference between two successive rays is

δ = $\frac{2\pi}{x}$(2d cosθ)
y = a sin ωt

Transmitted wave will be
y_1 = T sin ωt
y_2 = TR sin(ωt- δ)
y_3 = TR^2 sin (ωt- 2δ)
y_4 = TR^3 sin (ωt- 3δ)

Final resultant vibration can be written as D sin (ωt- φ)
D = instantaneous resultant amplitude
φ = resultant phase

Resultant vibration will be sum of partial vibration. According to superposition of wave motion,

D sin (ωt- φ) = T sin ωt + TR sin(ωt- δ) + TR^2 sin (ωt- 2δ) + TR^3 sin (ωt- 3δ) +………

Expanding sin terms and equating the coefficients of sin ωt and cos ωt,

D sin ωt. cosφ - D cos ωt. sin φ = T sin ωt + TR sin(ωt- δ) + TR^2 sin (ωt- 2δ) + TR^3 sin (ωt- 3δ) +………

D cosφ = T + TR cos δ + TR^2 cos 2δ +……..
D sin φ = TR sin δ + TR^2 sin 2δ +……………..

Resultant intensity, I = D^2
I = (D cosφ + i D sin φ) (D cosφ - i D sin φ)
Where i = $\sqrt{-1}$

I = $\left(\frac{T}{1-Re^{i\delta}}\right)\left(\frac{T}{1-Re^{-i\delta}}\right)$

Solving this we get,

$$I = \frac{T^2}{(1-R)^2[1 + Fsin^2\frac{\delta}{2}]}$$

F = $\frac{4R}{(1-R)^2}$

Resultant transmitted intensity varies with phase δ and reflection coefficient R.

It will be maximum at

$$sin^2\frac{\delta}{2} = 0$$

$$\frac{\delta}{2} = n\pi$$

$\delta = 2n\pi$,   where n= 0,1,2,……

$I_{max} = \frac{T^2}{1-R^2}$

It will be minimum at

$$sin^2\frac{\delta}{2} = +1, -1$$

$$sin\frac{\delta}{2} = 1$$

$$\frac{\delta}{2} = (2n+1)\frac{\pi}{2}$$

$$\delta = (2n+1)\pi$$

$I_{min} = \frac{T^2}{(1+R)^2}$