Arithmetic Progression

by

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Definition :– Arithmetic progression is a sequence or series of numbers arranged in either ascending or descending order such that the difference between any two consecutive terms is constant through out the series. This constant difference is called common difference of arithmetic progression and is denoted by ‘ d’.

For example the sequence 3 , 5 , 7 , 9 , 11 , 13……….is an arithmetic progression with common difference $d = (5 - 3) = (7 - 5) = (9 - 7)\ldots\ldots\ldots = 13 - 11 = 2$

Terms used in arithmetic progression and its symbol of presentation :-

For any sequence or series to be identified as arithmetic progression the minimum number of terms must be at least 3 (three). Maximum number of terms may be infinite.

Following are the symbols normally used to represent the A.P.

(1) First term of an A.P. is represented by "a" or " T_1 " .

(2) Common difference of the A.P. is presented by " d ".

(3) Numbers of terms of an AP is presented by " n".

(4) Value of nth term is presented by "Tn ".

(5) Sum of the series up to "n" terms = Sn.

Derivation of formula for finding nth term (general term) of an A.P. :-

Let ‘a’ is the first term and ‘d’ is the common difference of an A.P. Let ‘n’ is number of terms and ‘Tn’ is the value of nth term. There as per definition :-

First term of A.P = T_1 = a

2nd term of the A.P = T_2 = a + d

3rd term of the A.P = T_3 = a + 2d

4rth term of the A.P = T_4 = a + (4 – 1)d

(To obtain symmetry both side)

nth term = Tn = a + (n - 1)d……….(1)

This is formula for finding value of nth term of an A.P.

Derivation of formula for finding sum up to ‘n’ terms of an A.P :-

Let ‘a’ is the first term ‘d’ is the common difference and ‘n’ is number of terms of an A.P. Let Sn is sum of the A.P up to ‘n’ terms. Let ‘l’ is the last term of the A.P which is equal to nth term Tn. Thus the A.P can be written as

$S_n = a + (a + d) + (a + 2d) + \ldots\ldots\ldots + (l - 2d) + (l - d) + l \ldots\ldots\ldots\ldots (1)$

Now writing the series in reverse way

$S_n = l + (l - d) + (l - 2d) + \ldots\ldots\ldots + (a + 2d) + (a + d) + a \ldots\ldots\ldots\ldots (2)$

Now adding both series term wise.

$2Sn = (a + l) + (a + l) + (a + l)\ldots\ldots\ldots\ldots + (a + l) + (a + l) + (a + l)$

so ,

$2Sn = (a + l)$ summed 'n' times

$2Sn = n(a + l)$

Sn = n/2((a + l)

We also know that $l = Tn = a + (n – 1)d$. putting the value of $l = a + (n – 1)d$ in equation (2) we get,

$Sn = n/2[a + a + (n – 1)d]$

$Sn = n/2[2a + (n – 1)d]$……………….(3)

Example :–

Q:=> Which term of the A.P

49 , 44 , 39 , ……………….is 9 ?

Sol : Let nth term Tn = 9

given a = 49 , d = -5 n = ?

So,

$$T_n = a + (n - 1)d$$

$$9 = 49 + (n - 1)(-5)$$

or $9 - 49 = -5n + 5$

or $-40 = -5n + 5$

$$-45 = -5n$$

$$n = -45/-5 = 9$$

$$T_9 = 9$$

Example:–

Q:=> Find sum of the series ¾+2/3+7/12 ……up to 19 terms

Given: a = ¾, d = 2/3 – ¾

8-2/12 = -1/12

n = 19.

so,

S_{19} = n/2[2a + (19 – 1)d]

= 19/2[2*3/4 + 18(-1/12)]

= 19/2[3/2 – 3/2]

= 19/2*0 = 0

S_{19} = 0

Thank you