POWER POINT PRESENTATION [PPT]

By

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CONTENTS

CHAPTER – 1
HIGHER ORDER DERIVATIVES

1.1 INTRODUCTION

In this chapter we shall learn to derive higher orders derivatives of a given function successively. For this reason it is also known as successive differentiation.

1.2 SOME NOTATION

Given a function $y = f(x)$ we denote $y_1, y_2, y_3, \ldots. y_n$ as the first, second, third and the nth derivatives of $y = f(x)$ respectively.

1.3 The nth order derivations of some standard functions :

To find the nth derivative of the following functions:

I $$y = e^{mx}$$

$$y_1 = me^{mx}$$

$$y_2 = m^2 e^{mx}$$

$$y_3 = m^3 e^{mx}$$

.

.

$$y_n = m^n e^{mx}$$

Cor: Given that $y = a^{mx}$

$$y = e^{mxloga} e^{(mlog\, a)} x$$

$$\Rightarrow \quad y_n = (m\, loga)^n \mathrm{e}^{(mloga)\, x}$$

$$y_n = (m\, loga)^n a^{mx}$$

II $$y = (ax+b)^m$$

$$y_1 = m(ax+b)^{m-1} . a$$

$$y_2 = m(m-1)(ax+b)^{m-2} . a^2$$

$$y_3 = m(m-1)(m-2)(ax+b)^{m-3} . a^3$$

.

.

$$y_n = m(m-1)(m-2)\ldots[m-(n-1)](ax+b)^{m-n} . a^n$$

Cor : If m = n

Then

$$y_n = n(n-1)(n-2)\ldots 3.2.1.\,a^n$$

$\Rightarrow$ $y_n = a^n.\underline{n}$

$\Rightarrow$ $y_{n+1} = 0$

$\Rightarrow$ $y_{n+2} = 0$ and so on.

III. $y = \dfrac{1}{ax+b}$

Given $y = \dfrac{1}{ax+b} = (ax+b)^{-1}$

$$y_1 = (-1)(ax+b)^{-2}.\,a$$

$$y_2 = (-1)(-2)(ax+b)^{-3}.\,a^2$$

$$y_3 = (-1)(-2)(-3)(ax+b)^{-4}.\,a^3$$

.

.

$$y_n = (-1)(-2)(-3)\ldots(-n)(ax+b)^{-(n+1)}.\,a^n$$

$$\frac{a^n(-1)^n\underline{n}}{(ax+b)^{n+1}}$$

Cor: If a = 1 Then $y = \dfrac{1}{ax+b}$

Then $y_n = \dfrac{(-1)^n\underline{n}}{(x+b)^{n+1}}$

IV. $y = \sin(ax+b)$

$$y_1 = a\cos(ax+b) = a\sin\left(\frac{\pi}{2}+ax+b\right)$$

$$y_2 = a^2\cos\left(\frac{\pi}{2}+ax+b\right) = a^2\sin\left(\frac{2\pi}{2}+ax+b\right)$$

.

.

$$y_n = a^n\sin\left(\frac{npi}{2}+ax+b\right)$$

V. $y = \cos(ax+b)$

$$y_1 = -a\sin(ax+b) = a\cos\left(\frac{\pi}{2}+ax+b\right)$$

$$y_2 = +a^2\cos\left(\frac{\pi}{2}+ax+b\right)$$

.

.

$$y_n = +a^n\cos\left(\frac{n\pi}{2}+ax+b\right)$$

VI. $y = e^{ax}\sin(bx+c)$

$$y_1 = ae^{ax}\sin(bx+c) + be^{ax}\cos(bx+c)$$

$$= e^{ax}[a\sin(bx+c) + b\cos(bx+c)]$$

Put $a = r\cos\theta$ and $b = r\sin\theta$

$r^2 = a^2 + b^2$ and $\tan\theta = \frac{b}{a}$

Thus $y_1 = e^{ax}[r\cos\theta . \sin(bx+c) + r\sin\theta . \cos(bx+c)]$

$$= r e^{ax}[\sin(bx+c+\theta)]$$

Similary $y_2 = r^2 . r e^{ax}[\sin(bx+c+\theta+\theta)]$

$$= r^2 . r e^{ax}[\sin(bx+c+2\theta)]$$

$$y_3 = r^3 . e^{ax}[\sin(bx+c+3\theta)]$$

.

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$$y_n = r^n . e^{ax}[\sin(bx+c+n\theta)]$$

$$y_n = (a^2+b^2)^{\frac{n}{2}} e^{ax}[\sin(bx+c+n\tan^{-1}\frac{b}{a})]$$

VII. $y = e^{ax}\cos(bx+c)$

$$y_1 = ae^{ax}\cos(bx+c) - be^{ax}\sin(bx+c)$$

$$= e^{ax}[a\cos(bx+c) - b\sin(bx+c)]$$

Put $a = r\cos\theta$ and $b = r\sin\theta$

$r^2 = a^2 + b^2$ and $\tan\theta = \frac{b}{a}$

$$y_1 = r e^{ax}\cos(bx+c+\theta)$$

$$y_2 = r . r . e^{ax}\cos(bx+c+\theta+\theta)$$

$$y_2 = r^2 . e^{ax}\cos(bx+c+2\theta)$$

$$y_3 = r^3 . e^{ax}\cos(bx+c+3\theta)$$

.

.

$$y_n = r^n . e^{ax}\cos(bx+c+n\theta)$$

$$y_n = (a^2+b^2)^{\frac{n}{2}} e^{ax}\cos(bx+c+n\tan^{-1}\frac{b}{a})$$

VIII. $y = \log(ax+b)$

$$y_1 = \frac{1}{ax+b} . a = a(ax+b)^{-1}$$

$$y_2 = a^2(-1)(ax+b)^{-1}$$

$$y_3 = a^3(-1)(-2)(ax+b)^{-3}$$

.

.

$$y_n = a^n(-1)(-2).....[-(n-1)](ax+b)^{-n}$$

$$= \frac{a^n(-1)^{n-1}\underline{n-1}}{(ax+b)^n}$$

Example :

If $y=\sin mx+\cos mx$ Then Show that

$$y_n = m^n[1+(-1)^n \sin 2mx]^{\frac{1}{2}}$$

Solution : Given that

$$y = Sinmx + Cosmx$$

$$y_n = m^n \sin(mx+\frac{n\pi}{2}) + m^n \cos(mx+\frac{n\pi}{2})$$

$$y_n = m^n[\sin(mx+\frac{n\pi}{2}).\cos(mx+\frac{n\pi}{2})^2]^{\frac{1}{2}}$$

$$y_n = m^n[1+2\sin(mx+\frac{n\pi}{2}).\cos(mx+\frac{n\pi}{2})]^{\frac{1}{2}}$$

$$y_n = m^n[1+\sin 2(mx+\frac{n\pi}{2})]^{\frac{1}{2}}$$

$$y_n = m^n[1+\sin(2mx+n\pi)]^{\frac{1}{2}}$$

$$y_n = m^n[1+\sin 2mx.Cosn\pi + \cos 2mx.\sin n\pi]^{\frac{1}{2}}$$

$$y_n = m^n[1+(-1)^n \sin 2mx]^{\frac{1}{2}}$$

Since $Cosn\pi = 1$ if n is even

= - 1 if n is odd

So $\cos n\pi = (-1)^n$

and $\sin n\pi = 0$

1.4 **Leibnitz's Theorem**

If u and v are functions of x possessing derivates of the nth order Then,

$$(uv)_n = n_{C_0} u_n v + n_{C_1} u_{n-1} v + n_{C_2} u_{n-2} v_2 + ... + n_{C_{n-1}} u_1 v_{n-1} + n_{C_n} u v^n$$

Proof :

The proof is given by the principle of mathematical induction on n.

Let y = uv

Step 1 Take n = 1 then

$$y_1 = u_1 v + u v_1 = 1_{C_0} u_1 v + 1_{C_1} u v_1$$

The theorem is true for n = 1

Step 2. We assume that the theorem is true for n = m.

So, $(uv)_m = m_{C_0} u_m v + m_{C_1} u_{m-1} v_1 + m_{C_2} u_{m-2} v_2 + \ldots + m_{C_{m-1}} u_1 v_{m-1} + m_{C_m} u v_m$

Differentiating we get

$$(uv)_{m+1} = m_{C_0}(u_{m+1} v + u_m v_1) + m_{C_1}(u_m v_1 + u_{m-1} v_2) + m_{C_2}(u_{m-1} v_2 + u_{m-2} v_3) + \ldots.$$

$$\ldots m_{C_{m-1}}(u_2 v_{m-1} + u_1 v_m) + m_{C_m}(u_1 v_m + u v_{m+1})$$

$$m_{C_0} u_{m+1} v + (m_{c_0} + m_{c_1}) u_m v_1 + (m_{c_1} + m_{c_2}) u_{m-1} v_2 + \ldots.(m_{c_{m-1}} + m_{c_1}) u_1 v_m + (m_{c_m} u v_{m+1})$$

$$m+1_{C_0} u_{m+1} v + m+1_{C_1} u_m v_1 + m+1_{C_2} u_{m-1} v_2 + \ldots m+1_{C_m} u_1 v_m + m+1_{C_{m+1}} u v_{m+1}$$

Note that (i) $m_{c_{r-1}} + m_{c_r} = m+1_{c_r}$

(ii) $m_{c_0} = 1 = m+1_{c_0}$

(iii) $m_{c_m} = 1 = m+1_{c_{m+1}}$

Therefore, the theorem is true for n = m+1. Hence by Mathematical induction the theorem is true for any positive integer n.

1.5 Theorem : To prove that

$D^n\{e^{ax} f(x)\} = e^{ax}(D+a)^n f(x)$ where $D = \frac{d}{x}$

Proof : By Leibnitz's Theorem, we get

$$D^n\{e^{ax} f(x)\} = e^{ax}(D)^n f(x) + n_{c_1} a e^{ax} D^{n-1} f(x) + \ldots. n_{c_n} a^n e^{ax} f(x)$$

$$e^{ax}[((D)^n + n_{c_1} a D^{n-1} + \ldots. a^n) f(x)]$$

$$e^{ax}[(D+a)^n f(x)]$$

1.6 Applications of Leibnitz's Theorem to problems

Example 1 If $y = \sin(m \sin^{-1} x)$ Then Show that

i) $(1-x^2) y_2 - x y_1 + m^2 y = 0$

ii) $(1-x^2) y_{n+2} - (2n+1) x y_{n+1} + (m^2 - n^2) y_n = 0$

Solution : Given that

$$y = \sin(m \sin^{-1} x)$$

$$\Rightarrow \qquad \sin^{-1} y = m \sin^{-1} x$$

$$\Rightarrow \qquad \frac{1}{\sqrt{1-y^2}}\, y_1 = m . \frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow \qquad \sqrt{1-x^2}\, y_1 = m\sqrt{1-y^2}$$

$$\Rightarrow \qquad \sqrt{1-x^2}\; y_1^2 = m^2(1-y^2)$$

$$\Rightarrow \qquad (1-x^2).2y_1 y_2 + y_1^2(-2x) = m^2(-2y\,y_1)$$

$$\Rightarrow \qquad (1-x^2)\,y_2 - x\,y_1 + m^2 y = 0$$

i) Proved

For ii), Differentiating n times by Leibnitz's theorem

$$y_{n+2}(1-x^2) + n_{c_1} y_{n+1}(-2x) + n_{c_2} y_n(-2) - y_{n+1}x - n_{c_1} y_n + m^2 y_n = 0$$

$$(1-x^2)\,y_{n+2} + n\,y_{n+1}(-2x) + \frac{n(n-1)}{\underline{2}}\,y_n(-2) - x\,y_{n+1}x - n\,y_n + m^2 y_n = 0$$

$$(1-x^2)\,y_{n+2} - (2n+1)\,x\,y_{n+1} + (m^2 - y^2)\,y_n = 0$$

ii) Proved

CHAPTER – 2
HYPERBOLIC FUNCTIONS

2.1 Defintiions

I. Hyperbolic Sine Function

The function $\frac{e^x - e^{-x}}{2}$, where x is real or complex number, is called hyperbolic sine function and denoted by Sinhx

Thus $\sin hx = \frac{e^x - e^{-x}}{2}$

II. Hyperbolic Cosine function

The function $\frac{e^x + e^{-x}}{2}$ is called hyperbolic Cosine function of x and is denoted by Cos hx.

Thus $\cos hx = \frac{e^x + e^{-x}}{2}$

The hyperbolic tangent, cotangent, secant and cosecant of x are denoted and defined by the following relations :

$$\tan hx = \frac{\sin hx}{\cos hx} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\cot hx = \frac{\cos hx}{\sin hx} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

$$Sec\, hx = \frac{1}{\cos hx} = \frac{2}{e^x + e^{-x}}$$

$$Cosec\, hx = \frac{1}{\sin hx} = \frac{2}{e^x - e^{-x}}$$

Note i) $e^x = Coshx + Sinhx$

and $e^{-x} = Coshx - Sinhx$

ii) $\sin h0 = \frac{e^0 - e^{-0}}{2} = 0$

and $\cos h0 = \frac{e^0 + e^{-0}}{2} = 1$

3.2 Relation between circular functions and hyperbolic functions.

I. To show that

$$\sin ix = i \sin hx$$

$$\cos ix = i \cos hx$$

$$\tan ix = i \tan hx$$

Where $i = \sqrt{-1}$

By definition, we know that

$$\sin x = \frac{e^{ix} - e^{-x}}{2i}$$

Putting ix for x

$$\sin ix = \frac{e^{i(ix)} - e^{-i(ix)}}{2i} = \frac{e^{-x} - e^{x}}{2i} = i\, sinhx$$

Again $\cos x = \frac{e^{ix} + e^{-ix}}{2}$

$$\cos ix = \frac{e^{i(ix)} + e^{-i(ix)}}{2}$$

$$= \frac{e^{-x} + e^{x}}{2} = Coshx$$

$$\tan ix = \frac{\sin ix}{\cos ix} = \frac{i \sin hx}{\cos hx} = i \tan hx$$

Similary other relations can be obtained.

2.3 **Fundamental formulae :**

I. To show that

$$\cos h^2 x - \sin h^2 x = 1$$

$$sec\, h^2 x - \tan h^2 x = 1$$

$$\cot h^2 x - cosec\, h^2 x = 1$$

$$\cos h^2 x - \sin h^2 x = (\frac{e^x + e^{-x}}{2})^2 - (\frac{e^x - e^{-x}}{2})^2$$

$$= \frac{1}{4}\{e^{2x} + e^{-2x} + 2 - e^{2x} - e^{-2x} + 2\}$$

$$= \frac{1}{4}.4 = 1$$

Thus $\cos h^2 x - \sin h^2 x = 1$

Dividing by $\cos h^2 x$ we get

$$sec\, h^2 x + \tan h^2 x = 1$$

Again Dividing by $\sin h^2 x$ we get

$$\cot h^2 x - cosec\, h^2 x = 1$$

II. $\cosh(-x) = \cos hx$

$\sinh(-x) = -\sin hx$

By defintion $\cosh(-x) = \frac{e^{(-x)} + e^{-(-x)}}{2} = \cos hx$

and $\sin h(-x) = \frac{e^{(-x)} + e^{-(-x)}}{2} = -\sin hx$

III. $\cosh(x+y) = \cos hx . Coshy + Sinhx . Sinhy$

$\sinh(x+y) = \sin hx . Coshy + coshx . Sinhy$

We know that

$$\cosh(x+y) = \cos i(x+y)$$
$$= \cos(ix+iy)$$
$$= \cos ix . \cos iy - \sin ix . \sin iy$$
$$\cosh(x+y) = coshx . \cos hy + \sin hx . sinhy$$
$$i\sinh(x+y) = \sin i(x+y)$$
$$= \sin(ix+iy)$$
$$= \sin ix . \cos iy + \cos ix . \sin iy$$
$$\sin h(x+y) = sinhx . \cos hy + \cos hx . sinhy$$

Similarly, $\tanh(x+y), \coth(x+y)$ etc are obtained.

These formulae are known as addition formula.

2.4 Expansions of Cos hx and Sin hx we have

$$Coshx = \cos(ix)$$
$$= 1 - \frac{(ix)^2}{2} + \frac{(ix)^4}{4} + \ldots \quad \text{to } \infty$$
$$= 1 + \frac{x^2}{2} + \frac{x^4}{4} + \ldots \quad \text{to } \infty$$
$$i\sin hx = \sin ix$$
$$= ix - \frac{(ix)^3}{3} + \frac{(ix)^5}{5} + \ldots \quad \text{to } \infty$$
$$\sin hx = x + \frac{x^3}{3} + \frac{x^5}{5} \quad \text{to } \infty$$

2.5 Periods of Hyperbolic functions when n is an integer, then we have

$$\sin h(x+2n\pi i) = -isini(x+2n\pi i)$$
$$= -i(ix-2n\pi) = -i\sin ix = \sin hx$$
$$\cos h(x+2n\pi i) = -i\, cosi(x+2n\pi i)$$
$$\cos(ix-2n\pi) = \cos ix = \cos hx$$
$$\tan h(x+n\pi i) = -i\, tani(x+n\pi i)$$
$$= -i\tan(ix-n\pi) = \tan hx$$

Thus these functions are periodic. The period of Sin hx and Cos hx are 2 nπi and the period of tanhx is nπi.

2.6 Inverse Hyperbolic Functions

If Sin hz = w then z is called the inverse hyperbolic Since of w and is denoted by $\sin h^{-1} w$

Thus Sin hz = w ⇔ $z = \sin h^{-1} w$

Similary, we define $\cos h^{-1} w, \tan h^{-1} w$ etc.

2.7 Value of inverse Hyperbolic functions

I. To find the value of $\cos h^{-1} w$

Let $z = \cos h^{-1} w$

$\Rightarrow \quad w = \cos hz = \frac{e^z + e^{-z}}{2}$

$\Rightarrow \quad e^{2z} - 2w e^z + 1 = 0$

$\Rightarrow \quad e^z = \frac{2w \pm \sqrt{4w^2 - 4}}{2} = w \pm \sqrt{w^2 - 1}$

$\Rightarrow \quad z = \log\left(w \pm \sqrt{w^2 - 1}\right)$

But $w - \sqrt{w^2 - 1} = \frac{1}{w + \sqrt{w^2 - 1}}$

$\therefore \quad z = 2n\pi i \pm \log\left(w + \sqrt{w^2 - 1}\right)$

and known as the general value of $\cos h^{-1} w$

The principal value of

$$\cos h^{-1} w = \log\left(w + \sqrt{w^2 - 1}\right)$$

II. To find the value of $\sin h^{-1} w$

Let $z = \sin h^{-1} w$

$\Rightarrow \quad w = \sin hz = \frac{e^z - e^{-z}}{2}$

$\Rightarrow \quad e^{2z} - 2w e^z - 1 = 0$

$\Rightarrow \quad e^z = \frac{2w \pm \sqrt{4w^2 + 4}}{2} = w \pm \sqrt{w^2 + 1}$

$\Rightarrow \quad z = 2n\pi + \log\left(w + \sqrt{w^2 + 1}\right)$

or $\quad z = 2n\pi + \log\left(w - \sqrt{w^2 + 1}\right)$

But $w - \sqrt{w^2 + 1} = \frac{-1}{\left(w + \sqrt{w^2 + 1}\right)}$

$\therefore \quad \log\left(w - \sqrt{w^2 + 1}\right) = \log(-1) - \log\left(w + \sqrt{w^2 + 1}\right)$

$= \quad \pi i - \log\left(w + \sqrt{w^2 + 1}\right)$

Thus $\quad z = 2n\pi i + \log\left(w + \sqrt{w^2 + 1}\right)$

or $\quad z = (2n+1)\pi i - \log\left(w + \sqrt{w^2 + 1}\right)$

Thus $z = n\pi i + (-1)^n \log\left(w + \sqrt{w^2+1}\right)$

It is known as the general value of $\sin h^{-1} w$

The principal value of

$$\sin h^{-1} w = \log\left(w + \sqrt{w^2+1}\right)$$

III. To find the value of $\tan h^{-1} w$

Let $z = \tan h^{-1} w$

$\Rightarrow$ $$w = \tan hz = \frac{e^z - e^{-z}}{e^z + e^{-z}}$$

$\Rightarrow$ $$w = \frac{e^{2z} - 1}{e^{2z} + 1}$$

$\Rightarrow$ $$e^{2z} = \frac{1+w}{1-w}$$ [By Componendo and Dividendo]

$\Rightarrow$ $$2z = 2n\pi i + \log\frac{1+w}{1-w}$$

$\Rightarrow$ $$z = n\pi i + \frac{1}{2}\log\frac{1+w}{1-w}$$

Which is known as the general value of $\tan h^{-1} w$

The principle value of

$$\tan h^{-1} w = \frac{1}{2}\log\frac{1+w}{1-w}$$

Similary, the principal value of $\cot h^{-1} zv = \frac{1}{2}\log\frac{w+1}{w-1}$

$$sec\, h^{-1} zv = \log\frac{1+\sqrt{1-w^2}}{w}$$

$$cosec\, h^{-1} zv = \log\frac{1+\sqrt{1+w^2}}{w}$$

2.8 Relation between the inverse hyperbolic functions and inverse circular functions

If $z = \cos h^{-1} w$

$\Rightarrow \quad w = \cos hz$

$\Rightarrow \quad w = \cos iz$

$\Rightarrow \quad \cos^{-1} w = iz$

$\Rightarrow \quad z = -i\cos^{-1} w$

$\Rightarrow \quad \cos h^{-1} w = -i\cos^{-1} w$

Similary,

$$\sin h^{-1} w = -i\sin^{-1} w$$

$$\tan h^{-1} w = -i\tan^{-1} w$$

2.9 Some problems

Separate into real and imaginary parts

I. Cos h $(\alpha + i\beta)$

Solution Given

Cos h (α+iβ) = Cos i (α+iβ)

= Cos (iα - β)

= Cos iα . Cos β + Sin iα + Sin β

Cos h (α+iβ) = Cos hα . Cosβ + i Sin h α . Sin β

II. $\cos^{-1}(x+iy)$

Solution : Let $\cos^{-1}(x+iy) = A + iB - 0$

Then $\cos^{-1}(x-iy) = A - iB$

$$2A = \cos^{-1}(x+iy) + \cos(x-iy)$$

$$\cos^{-1}(x+iy).(x-iy) - \sqrt{1-(x+iy)^2}.\sqrt{1-(x-iy)^2}$$

$\Rightarrow \quad A = \frac{1}{2}\cos^{-1} x^2 + y^2 - \sqrt{(1-x^2+y^2)^2} + 4x^2y^2$

$= \quad 2iB = \cos^{-1}(x+iy) - \cos^{-1}(x-iy)$

$$\cos^{-1}[(x+iy).(x-iy) + \sqrt{1-(x+iy)^2}.\sqrt{1-(x-iy)^2}]$$

$$\cos i\,2B = x^2 + y^2 + \sqrt{(1-x^2+y^2)^2 + 4x^2y^2}$$

$$\cos h\,2B = x^2 + y^2 + \sqrt{(1-x^2+y^2)^2 + 4x^2y^2}$$

$$B=\frac{1}{2}\cos h^{-1}[x^2+y^2+\sqrt{(1-x^2+y^2)+4x^2y^2}]$$

Putting the value of A and B in equation (i) we get the required result.

III. If Sin(*θ*+i*Φ*) = *tan* (*x*+*iy*) *Then show that*

$$\frac{\tan Ø}{\tan \phi}=\frac{\sin 2x}{\sin 2y}$$

Solution : *Since*

Sin(*θ*+i*Φ*) = *tan* (*x*+*iy*)

∴ Sin(*θ*-i*Φ*) = *tan* (*x*-*iy*)

So

$$\frac{\sin(Ø+i\phi)}{\sin(Ø-i\phi)}=\frac{\tan(x+iy)}{\tan(x-iy)}$$

$$\Rightarrow \quad \frac{\sin(Ø+i\phi)+\sin(Ø-i\phi)}{\sin(Ø+i\phi)-\sin(Ø-i\phi)}=\frac{\tan(x+iy)+\tan(x+iy)}{\tan(x-iy)+\tan(x-iy)}$$

(*By componedo* & *dividends*)

$$\Rightarrow \quad \frac{2\sin Ø\,.\cos i\phi}{2\cos Ø\,.\sin i\phi}=\frac{\sin[(x+iy)+(x-iy)]}{\sin[(x+iy)-\sin(x-iy)]}$$

$$\Rightarrow \quad \frac{\tan Ø}{\tan i\phi}=\frac{\sin 2x}{\sin i\,2y}$$

Hence $\frac{\tan Ø}{\tan \phi}=\frac{\sin 2x}{\sin h\,2y}$

Thank

You

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