MOVING
CHARGES AND MAGNETISM

(e-content for UG)

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OBJECTIVE OF THE PRESENTATION

- **This is an e-content developed for UG students of Physics Core Semester-II (Paper Code: PHYCC203)**
- **The main objective is to provide the students a detailed study of the " Moving Charges and Magnetism ",under the paper Electricity and Magnetism .**
- **This initiative has been taken by Patna University under the guidelines given by UGC.**

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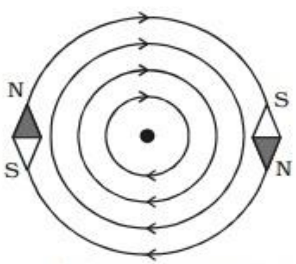
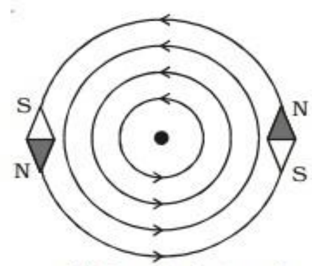
INTRODUCTION

- In 1820,Oersted dicovered that electricity and magnetism were intimately related . He experimentally showed that the electric current through the straight wire causes noticeable deflection of the magnetic compass needle held near the wire.
- He found that the magnetic needle is aligned tangentially to an imaginary circle which has the straight current carrying wire as its centre and has its plane perpendicular to the wire as shown in figure below . If the direction of current in the wire is reversed, the direction of the magnetic needle is also reversed as shown in the figure 1 below . The deflection increases on increasing the current in the wire or bringing the magnetic compass needle closer to the wire.

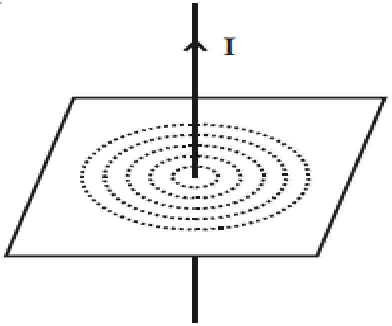
- He also found that the iron filings sprinkled around the wire , arrange themselves in concentric circles with the wire as centre in the plane perpendicular to the wire.
- He found that magnetic effect is created by flow of current.



(a) Current inwards



(b) Current Outwards



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MAGNETIC FIELD

- Electric charges in rest create an electric field namely the electrostatic field . Likewise ,electric charges in motion i.e. electric current creates a magnetic field.
- The space around a magnet or a current carrying conductor where magnetic force can be experienced is regarded as the magnetic field.
- The basic magnetic field vector $\vec{B}$ is called magnetic induction or magnetic flux density represented by the lines of induction.

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HOW TO DEFINE THE MAGNETIC FIELD B ?

- In the case of an electric field E, we have already seen that the field is defined as the force per unit charge:

$$E=F_e/q \qquad (1)$$

- However, due to the absence of magnetic monopoles, B must be defined in a different way.
- It is regarded as the magnetic effect of current and its definition is in terms of the additional force experienced by charged particle when there is magnetic field.

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- To define the magnetic field at a point, consider a particle of charge q and moving at a velocity ‘v’ . Experimentally we have the following observations:

1. The magnitude of the magnetic force exerted on the charged particle is proportional to both v and q.
2. The magnitude and direction of F_B depends on v & B
3. The magnetic force F_B vanishes when v is parallel to B . However, when v makes an angle θ with , the direction of F_B is perpendicular to the plane formed by B and v , and the magnitude F_B is proportional to $F_B \sin\theta$.
4. When the sign of the charge of the particle is switched from positive to negative (or vice versa), the direction of the magnetic force also reverses.

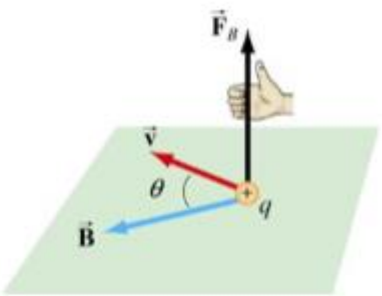
- This shows that the magnetic force

$$\vec{F}_B = q(\vec{v} \times \vec{B}) \qquad (2)$$

- The magnitude of force given by (2) is

$$F_B = qvB\sin\theta \qquad (3)$$

$$B = \frac{F_B}{qv\sin\theta}$$



- The above expression gives the magnitude of magnetic field and the direction is according to the right hand thumb rule.
- The S.I unit of magnetic field is Tesla(T) or Wb/square metre whereas the C.G.S unit is Gauss(G) or Maxwell/square cm.

$$1 \text{ Tesla} = 10^4 \text{ Gauss}$$

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Note that $\vec{\mathbf{F}}_B$ is always perpendicular to $\vec{\mathbf{v}}$ and $\vec{\mathbf{B}}$, and cannot change the particle's speed v (and thus the kinetic energy). In other words, magnetic force cannot speed up or slow down a charged particle. Consequently, $\vec{\mathbf{F}}_B$ can do no work on the particle:

$$dW = \vec{\mathbf{F}}_B \cdot d\vec{\mathbf{s}} = q(\vec{\mathbf{v}} \times \vec{\mathbf{B}}) \cdot \vec{\mathbf{v}}\, dt = q(\vec{\mathbf{v}} \times \vec{\mathbf{v}}) \cdot \vec{\mathbf{B}}\, dt = 0$$

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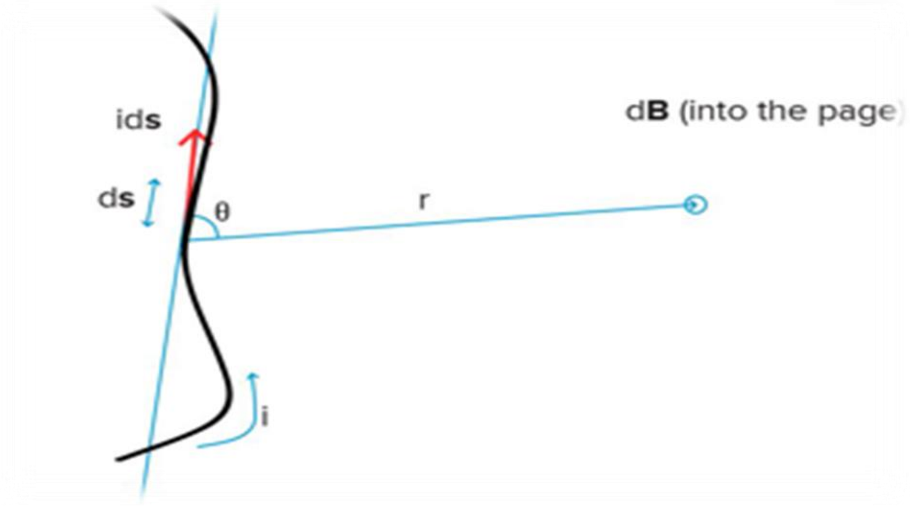
BIOT-SAVART'S LAW

- Biot-Savart's law is an equation that gives the magnetic field produced due to a current carrying segment. This segment is taken as a vector quantity known as the current element.



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- Consider a current carrying wire ‘i’ in a specific direction as shown in the above figure. Take a small element of the wire of length ds. The direction of this element is along that of the current so that it forms a vector i ds.
- To know the magnetic field produced at a point due to this small element, one can apply Biot-Savart’s Law. Let the position vector of the point in question drawn from the current element be r and the angle between the two be θ. Then,

$$|dB| = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{Idl\sin\Theta}{r^2}\right) \quad (4)$$

- Where $\mu_0 = 4\pi \times 10^{-7}$ is the magnetic permeablility of free space.

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BIOT-SAVART'S LAW VERSUS COULOMB'S LAW

➢ **Similarities**:

- Both magnetic and electric fields at a point are inversely proportional to the square of the distance between the field source and the point in question i.e both follow inverse square law.
- Electric field due to a point charge (Coulomb's law) is :

$$E = (1/4\pi\varepsilon_o) \times (q/r^2) \qquad (5)$$

- Magnetic field due to a moving charge (Biot-Savart law) is:

$$B = (\mu_o/4\pi) \times idl(\sin\theta)/r^2 \qquad (6)$$

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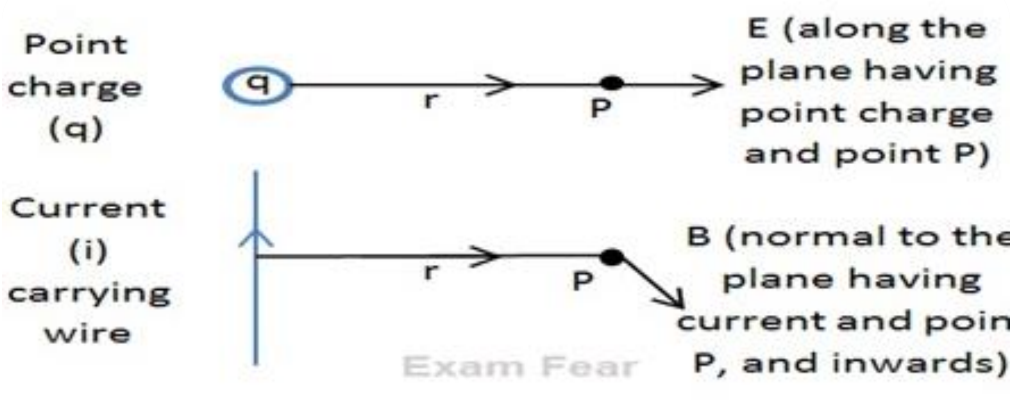

- Both laws works on the principle of superposition (resultant field due to more than 1 sources is the vector sum of all the sources independently)
- Both magnetic and electric fields have sources that are linear in nature (both, the current element idl and the electrostatic charge q)

- **Differences**:
- The source of electrostatic field is scalar in nature. Whereas, the source of magnetic field, which is current element (idl), is vector in nature.
- Electric field always acts along the plane containing distance (r) between point charge and the point where electric field is to be calculated. But, the magnetic field acts in the plane perpendicular to the plane of distance(r) between the current element and the concerned point.
- Magnetic field depends on the angle (θ) between the current element(idl) and line joining the point and current element. However, electric field doesn't depend on angle(θ).

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APPLICATIONS OF BIOT-SAVART'S LAW

- We can use Biot–Savart's law to calculate magnetic responses even at the atomic or molecular level.
- It is also used in aerodynamic theory to calculate the velocity induced by vortex lines.
- Biot-Savart's law is similar to the Coulomb's law in electrostatics.
- The law is applicable for very small conductors too which carry current.
- The law is applicable for symmetrical current distribution.

MAGNETIC FIELD DUE TO A STRAIGHT CURRENT CARRYING WIRE

- According to the Biot-Savart law, magnetic field dB at point P due to current element idl in the diagram is given by:

  $\therefore B = \int(\mu_o/4\pi)idl\cos\theta\,/x^2 = (\mu_o/4\pi)\int idl\cos\theta\,/x^2$ (i)

  $dB = (\mu_o/4\pi)idl\,\sin(90°-\theta)\,/x^2 = (\mu_o/4\pi)idl\cos\theta\,/x^2$

- Considering triangle ABN: $\cos\theta = AN/dl$

  $AN = dl\cos\theta$

- Considering triangle ANP: $\sin(d\theta) \tilde{} d\theta = AN/x$

  $AN = x(d\theta)$

- Using the value of AN from the above 2 equations:

  $dl\cos\theta = xd\theta$ (ii)

- Considering triangle AOP: $\cos\theta = r/x$

  $\therefore x = r/\cos\theta$ (iii)

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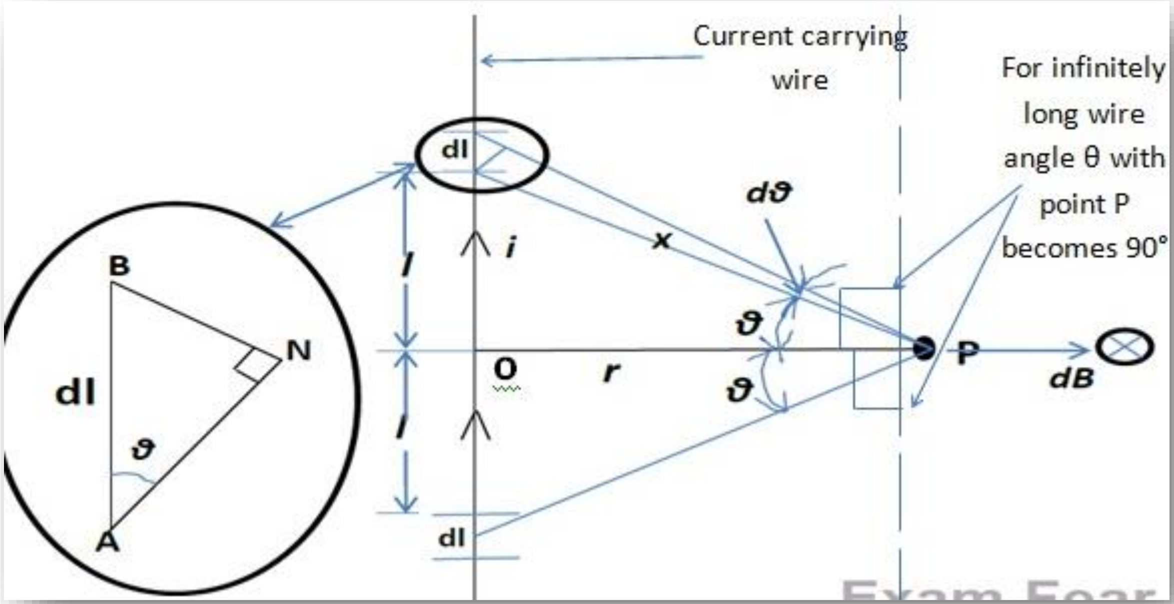
Using the values of dlcosθ from eq.(ii) and x from eq.(iii) in eq.(i):

$$B = \int(\mu_o/4\pi)I\ x\ d\theta/x^2 = \int(\mu_o/4\pi)id\theta/x = \int(\mu_o/4\pi)i(\cos\theta)dx/r$$

$$\therefore B = (\mu_o/4\pi)(\sin\theta_2 + \sin\theta_1)$$

For infinitely long wire ($\theta_1 = 90°$, $\theta_2 = 90°$):The above equation becomes

$$\therefore B = \mu_o i/(2\pi r)$$



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MAGNETIC FIELD ON THE AXIS OF A CIRCULAR CURRENT LOOP:

- Magnetic field dB at point P due to current element idl, making right angle to the line joining point P and current element, will be given by Biot-Savart law as:

$$dB = (\mu_o/4\pi)idl\sin(90°)/r^2 = (\mu_o/4\pi)idl/r^2$$

As we can see in the diagram, the magnetic field dB will have 2 component, i) the vertical component dB cosθ, and ii) the horizontal component dB sinθ

- It is also evident from the diagram that the vertical component dB cosθ will be cancelled by the equal and opposite component due to current element at the opposite of the above current element (due to symmetry).

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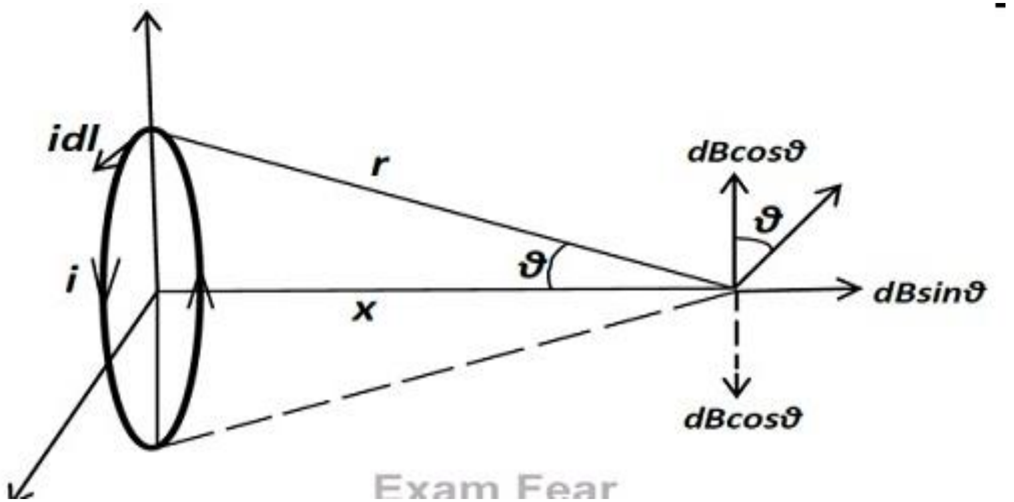

So, the total magnetic field will only be due to the horizontal component (dBsinθ) along the positive x-axis

$$dB\sin\theta = (\mu_o/4\pi)idl(\sin\theta)/r^2$$

$$\sin\theta = R/r = R/\sqrt{(x^2 + R^2)}$$

$$\therefore dB\sin\theta = (\mu_o/4\pi)iRdl/(x^2 + R^2)^{3/2}$$

So, the total magnetic field will be: $B = \dfrac{\mu_0 i R^2}{2(R^2 + x^2)^{3/2}}$

For magnetic field at the center of current loop x = 0 in the above expression.

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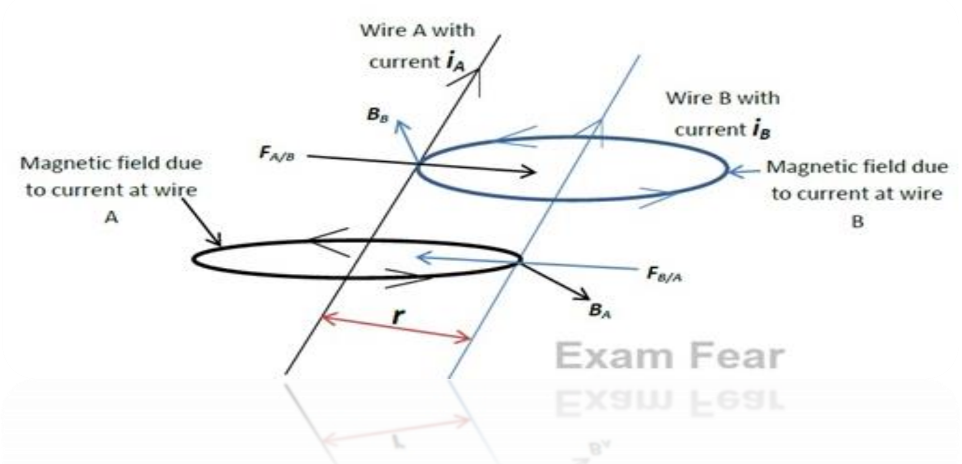
FORCE BETWEEN TWO PARALLEL CURRENT CARRYING WIRES

- **Case-1: Parallel wire carrying currents in the same direction**



When 2 parallel, current carrying wires are placed at some distance r from each other, they will experience a force on each other, due to magnetic field produced by each other.

- Let the wires A and B, carrying current iA and i_B respectively are placed r distance apart and parallel to each other

- Magnetic field(BB) at any point on wire A due to current i_B on wire B will be perpendicular to the plane having both wires and in the upwards direction (direction given by right hand thumb rule)

$$B_B = \mu_o i_B/(2\pi r)$$

- Force per unit length ($F_{A/B}$)on wire A due to magnetic field (B_B) produced by wire B will be given by:

$$F_{A/B} = i_A B_B \sin 90°$$
$$= i_A B_B$$

- $\therefore F_{A/B} = \mu_o i_A i_B/(2\pi r)$

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- The direction of $F_{A/B}$ will be perpendicular to wire A and towards wire B (by using Fleming's right hand thumb rule to find cross product direction, already discussed)
- Magnetic field(B_A) at any point on wire B due to current i_A on wire A will be perpendicular to the plane having both wires and in the downwards direction (direction given by right hand thumb rule)

$$B_A = \mu_o i_A/(2\pi r)$$

- Force per unit length ($F_{B/A}$) on wire B due to magnetic field (B_A) produced by wire A will be given by:

$$F_{B/A} = i_B B_A \sin 90° = i_B B_A$$

$$\therefore F_{A/B} = \mu_o i_A i_B/(2\pi r)$$

- The direction of $F_{B/A}$ will be perpendicular to wire B and towards wire A (by using Fleming's right hand thumb rule to find cross product direction, already discussed):

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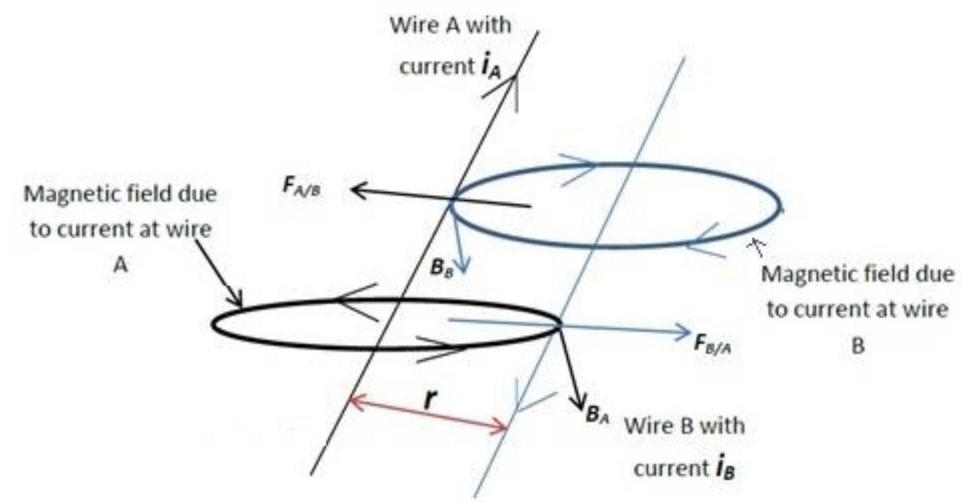
- Both the forces $F_{A/B}$ and $F_{B/A}$ are equal and towards each other, meaning the both the current carrying wires will be attracted towards each other

- In the above equation of force F, putting $i_A = i_B =$ 1A,and r = 1m

$$F = \mu_o \times 1 \times 1/(2\pi \times 1)$$
$$= \mu_o/(2\pi)$$
$$= 4\pi \times 10^{-7}/(2\pi)$$
$$\therefore F = 2 \times 10^{-7} N/m$$

- So, we can define 1Ampere current as the amount of current flowing in the 2 parallel straight wires placed 1meter apart, when the force acting on each wire per unit length is equal to the 2×10^{-7}N/m

PARALLEL WIRE CARRYING CURRENTS IN THE OPPOSITE DIRECTION



- On proceeding in the similar manner as the first case, we will find that the values of forces will be the same, only their directions get reversed (refer the diagram above)
- The forces will be equal but this time away from each other, i.e., the wires will move away from each other (repel each other)

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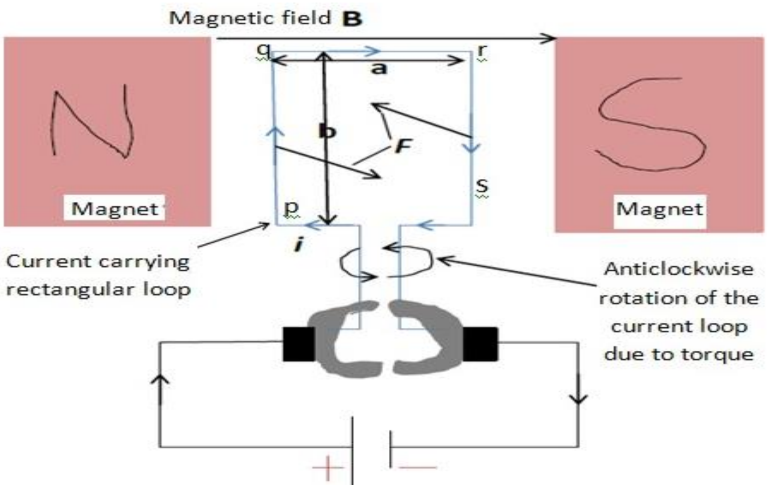
TORQUE ON A CURRENT LOOP:

- We already know that a current carrying wire experiences a force when placed in external magnetic field (in direction other than the magnetic field lines).
- Here is what happens when a current loop is placed in the external magnetic field



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A rectangular loop of vertical dimension b and horizontal dimension a, carrying current i , is placed in an external magnetic field B as shown below:

Force on a current i carrying conductor of length l making an angle θ with the external magnetic field B is given by:

$$F = ilB\sin\theta \qquad (7)$$

the net force on the current loop pqrs will be:

$$F_{net} = F_{pq} + F_{qr} + F_{rs} + F_{sp} = i_bB + 0 + (-i_bB) + 0$$

$$\therefore \mathbf{F_{net} = 0}$$

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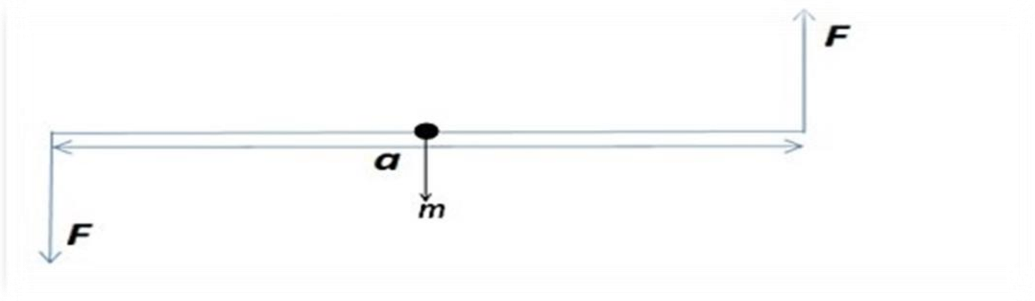
- Hence, no net force acting on the rectangular loop
- But the force on pq and rs are equal and opposite and acting on 2 different points of a body so together they constitute a couple. Hence, the net torque τ on the loop pqrs due to couple will be:

$$\tau = F \times a = i(ba)B = iAB = mB \qquad (8)$$

- Here, A = ab = area of the loop pqrs, and m = magnetic moment = iA(having direction same as the area vector of the loop, i.e. perpendicular to the plane of paper outwards)



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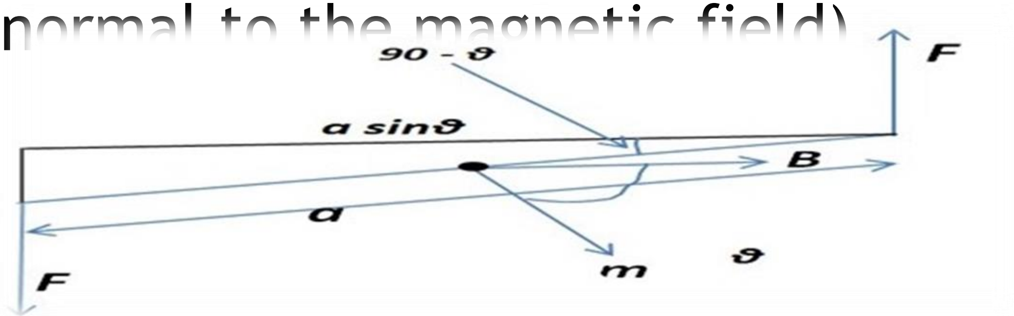
If the plane of the loop is rotated by angle θ in clockwise direction from bottom, then the area vector (or magnetic moment vector) would make an angle θ with the magnetic field lines, then the torque would be given by:

$$\tau = F \times a\sin\theta = i(ba)B\sin\theta = iAB\sin\theta = mB\sin\theta \quad (9)$$

The above equation shows that torque depends on the angle θ between magnetic moment and magnetic field. Torque will be maximum when θ = 90° (magnetic moment is normal to the magnetic field)



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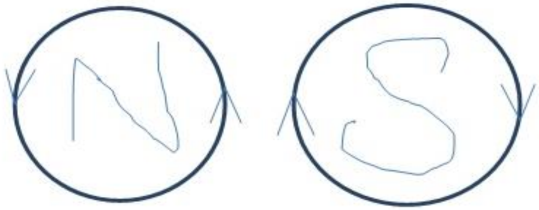
- Torque will be minimum ($\tau = 0$) when $\theta = 0^\circ$ or 180° (magnetic moment is parallel or antiparallel to the magnetic field). In this case, the system is said to be in equilibrium.
- In the case of $\theta = 0^\circ$, the system will be in stable equilibrium, i.e. if the loop is given a small angular displacement, the loop will come back to the initial position and angle will be 0 again
- In the case of $\theta = 180^\circ$, the system is said to be in the unstable equilibrium, i.e. if the loop is given a small angular displacement, the angular displacement will increase further.



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- For N number of turns, the magnetic moment will be given by:-

$$m = NiA$$

- The above equation of torque on a loop in a magnetic field is comparable to the torque on a dipole in an electric field.

- Hence, we can see that current loop behaves as a magnetic dipole with when viewed as anticlockwise current loop representing north pole, and when viewed as clockwise current loop representing south pole.

DIVERGENCE AND CURL OF THE MAGNETIC FIELD

- The static electric field E(x, y, z) — such as the field of static charges obeys equations

$$div\vec{E} = \frac{\rho}{\varepsilon_0} \qquad (10)$$

$$curl\vec{E} = 0 \qquad (11)$$

The static magnetic field B(x, y, z) — such as the field of steady currents obeys different equations

$$div\vec{B} = 0 \qquad (12)$$

$$curl\vec{B} = \mu_0 \vec{j} \qquad (13)$$

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- Due to this difference, the magnetic field of long straight wire looks quite different from the electric field of a point charge or a linear charge.
- The divergence of Magnetic field implies the non-existence of monopoles. It is also known as the Gauss's law of magnetism.
- The curl of magnetic field gives the Ampere's circuital law.

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THANK YOU!