

Geometric Progression

by

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Geometric progression

Definition :- The series and sequence in which every term is obtained by multiplying a constant number to previous term is called Geometric progression. The constant number denoted by 'r' is known as common ratio.

Thus common ratio of a G.P = $r = \text{any term} / \text{its previous term}$

If 'a' is the first term and 'r' is the common ratio of a G.P then we can obtain the G.P as below.

a, ar, ar², ar³, Up to n terms.

Thus here $T_2/T_1 = T_3/T_2 = T_4/T_3 =$
 $.....T_n/T_{n-1}$

To find formula for n^{th} term of a G.P

Let 'a' is the first term 'r' is the common ratio and
'n' is the number of terms of a G.P

Then

First term = $T_1 = a$

2nd term = $T_2 = ar$

3rd term = $T_3 = ar^2 = ar^{3-1}$ (To maintain symmetry)

4th term = $T_4 = ar^{4-1}$

nth term = $T_n = ar^{n-1}$

Formula for n^{th} term.

Example:-

Q:=> Find the 6th term of the series
8/5, 8/3, 40/9.....

In this sequence first term = a = 8/5

Common ratio = r = (8/3) / (8/5)

so, r = 5/3

$$T_6 = ar^{6-1}$$

$$8/5 (5/3)^{6-1}$$

$$= 8/5 * 5^5/3^5$$

$$= 5000/243$$

To find the sum of a G.P up to 'n' terms:-

Let 'a' is the first term and 'r' is the common ratio of a G.P. Let S_n represents the sum up to 'n' terms of the G.P.

so , n^{th} term = $T_n = ar^{(n-1)}$

$$S_n = a + ar + ar^2 + \dots + ar^{(n-1)} \quad \dots\dots\dots(1)$$

$$r * S_n = ar + ar^2 + \dots + ar^{n-1} + ar^n \quad \dots\dots(2)$$

(Multiplying (1) by r both side and shifting each term to one place.)

Now subtracting equation (2) from (1) we get

$$S_n(1-r) = a - ar^n = a(1-r^n)$$

$$S_n = a(1-r^n)/(1-r) \dots \dots \dots (3)$$

If we subtract equation (1) from equation (2)
we get

$$S_n = a(r^n-1)/(r-1) \dots \dots \dots (3)$$

Formula (2)

$S_n = a(1-r^n)/(1-r)$ is used if $r < 1$.

Formula (3)

$S_n = a(r^n-1)/(r-1)$ is used if $r > 1$

Example :-

Q:=> Find the sum of the series up to 5 term

$1/4 - 1/2 + 1 - 2 + 4 \dots\dots\dots$

Given $a = 1/4$, $r = (-1/2)/(1/4)$

$r = -2$ $n = 5$

$S_5 = a(1-r^n)/(1-r)$ [Since $r < 1$]

$$1/4[1-(-2)^5]/1-(-2)$$

$$= 1/4\{1+32\}/1+2$$

$$= 1/4*(33/3)$$

$$= 33/12$$

$$= 11/4$$

Thank you

The background features abstract, overlapping geometric shapes in various shades of green, ranging from light lime to dark forest green. These shapes are primarily located on the right side of the frame, creating a modern, layered effect against the white background.