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Study of non -classical properties in generalised Jaynes Cummings model

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Abstract: *Mathematical models that permit exact analytic solutions to idealized physical system have been vital to the development of science. The Jaynes-Cummings model (JCM) has a very significant role not only in many theoretical predictions but also in explanation of experiments in cavity-QED. The revival of interest in JCM is due to the observation of many non-classical effects such as collapse-revival phenomena, atom-atom entanglement etc.*

The system of two two-level atoms (TLA) inside a cavity has attracted considerable attention, both because it has become experimentally feasible and because it is paradigm to study the evolution of entanglement.

In the existing theoretical work a generalized model of standard JCM (Jaynes et al 1963) with two identical two-level atoms that resonantly interact with electromagnetic fields inside the cavity through three-photon three-mode process is studied. Cavity-QED generally deals with a few cavity photons. As such the addition (atomic emission) or annihilation (atomic absorption) effects are expected to change the atom field interaction strength significantly. Hence, the Hamiltonian can also depend on the intensity of the cavity field (s) with which it is interacting and it would be appropriate to extend and study the problems introducing an intensity–dependent coupling constant in the corresponding Hamiltonian. This is what we have considered in our analysis of problems related to cavity-QED.

We consider a model consisting of a lossless cavity through which two two-level atoms pass one after another. The first atom interacts with the cavity field via three photon process and leaves the cavity. The second atom then enters the cavity and interacts with the field with the changes made by its interaction with the first atom. It has been shown earlier that the two atoms get entangled in the process even though they do not interact directly. The properties of the radiation field encountered by them bears crucially on the nature of entanglement.

A unitary transformation method is used to solve the time-dependent problem that also gives the eigensolutions of the interacting Hamiltonian (Singh 2006). The state of the model has been calculated and then the atom –atom entanglement has been analyzed graphically.

Keywords: *Jaynes-Cummings Model, Entanglement,Two-level atom,Hamiltonian, T-Operator,Density Matrix.*

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Introduction :

The study of interaction between radiation and atoms is of significance because of different non-classical effects resulting such as collapse and revival phenomena, quantum entanglement non- local correlation functions.

This Interaction may cause in absorption or emission of photons resulting in single photon or multi-photon process. If the transition frequency of absorption / emission matches with the frequency of applied field then single photon process occurs known as linear process.

In the multi-photon process, an atom makes a transition from its ground state to an excited state by the simultaneous absorption of n- photons via non-resonant intermediate levels. The photons that are absorbed can have the same frequency (degenerate two photon process) or different frequency (non-degenerate two photon process).

The Jaynes-Cummings model (JCM) comprises a single two-level atom interacting with a single quantized mode of electromagnetic field in a high Q cavity under the Rotating Wave Approximation. Due to improvement in the cavity QED experiments in the last few decades, the Jaynes-Cummings model could be realized in laboratories. Because the dynamics predicted by the JCM has been supported by the experiment and such systems have been found to be suitable for studying purely quantum mechanical effects, there is intensive interest in the theoretical generalization of the JCM (Gerry et. al. 1990, Gerry et. al. 1992 , Datta et al 2004)

In the present model we have studied in the case of three mode three photon process i.e. when the transition between the two levels of an atom occurs by simultaneous absorption or emission of three photons. It is one of the generalized model of the standard Jaynes Cummings Model.

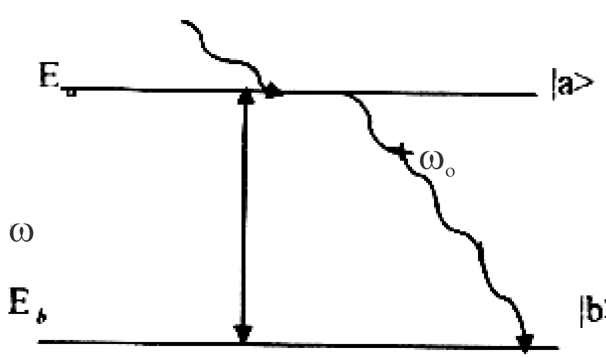

Fig. 1. Resonant emission if w = w$_o$

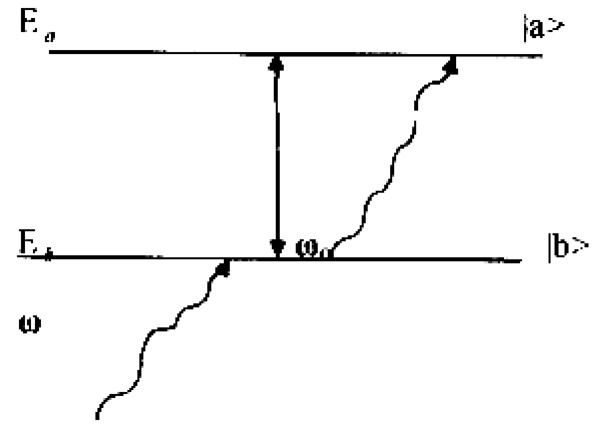

Fig. 2. Resonant Absorption

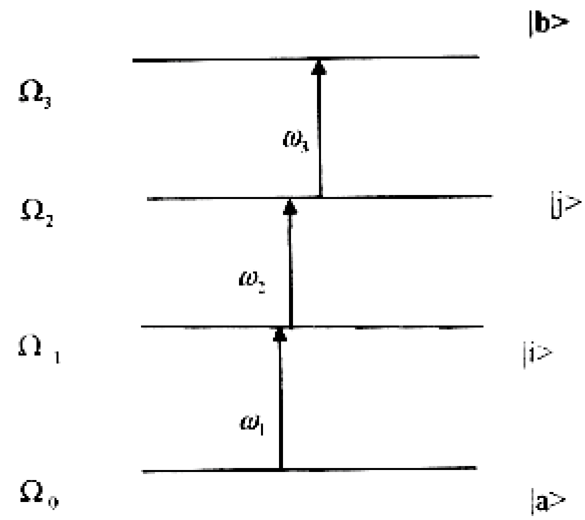

Fig. 3. Three photon absorption

Entanglement is one of the striking features of quantum mechanics that distinguishes quantum information theory from a classical one. It plays a key role in quantum information, quantum computation and quantum cryptography.

The simplest scheme to investigate the atom- field entanglement is the Jaynes –Cummings model (JCM) that describes the interaction of a two-level atom with a single mode quantized radiation field. The model is exactly solvable in the framework of the rotating wave approximation and is experimentally realized.

Two Atom Three Mode Jaynes Cummings Model: We consider a model consisting of a lossless cavity having high Q comprised of three modes of frequency ω_1, ω_2 and ω_3 through which two-level Rydberg atoms pass one after another. The first atom interacts with the cavity field as a result of which the cavity field statistics is changed. Since the cavity is lossless, the changes remain in an unaltered state until the second atom arrives at the cavity. Thus, the second atom interacts with the field with the changes made by its interaction with the first atom. It has been shown earlier that the two atoms get entangled in the process even though they do not interact directly (Singh et. al., 2013).

The non-degenerate three-photon Jaynes Cummings model is an effective three level atom interacting with three different modes, of the field. The Hamiltonian for the model for such a system in the rotating wave approximation is written as

$$\hat{H} = \hat{H}_0 + \hat{H}_i \tag{1}$$

$$\hat{H}_0 = \frac{\hbar\omega_0}{2}\hat{\sigma}_3 + \hbar\omega_1\hat{a}_1^\dagger\hat{a}_1 + \hbar\omega_2\hat{a}_2^\dagger\hat{a}_2 + \hbar\omega_3\hat{a}_3^\dagger\hat{a}_3 \tag{2}$$

$$\hat{H}_i = \hbar g(\hat{a}_1^\dagger\hat{a}_2^\dagger\hat{a}_3^\dagger\hat{\sigma}_- + \hat{\sigma}_-\hat{a}_1\hat{a}_2\hat{a}_3) \tag{3}$$

Where ω_0 is the transition frequency and g is the atom-field coupling constant, σ_3 is the inversion operator and σ_+, σ_- are the Pauli raising and lowering operators respectively. $\hat{a}_1^\dagger, \hat{a}_1$ and $\hat{a}_2^\dagger, \hat{a}_2, \hat{a}_3^\dagger, \hat{a}_3$ are the creation & annihilation operators of mode 1, 2 & 3.

A straightforward method, that of unitary transformation in quantum mechanics has been used to obtain the eigenfunctions and eigenvalues of the Hamiltonian of the interacting system in the two models. The method, besides being general is mathematically simpler.

The system we consider is an effective three -level atom with upper and lower states denoted by $|a\rangle$ and $|b\rangle$ respectively. In the three-photon processes, some intermediate states $|i\rangle$ &|j> are involved, which are assumed to be coupled to $|a\rangle$ and $|b\rangle$ by dipole-allowed transitions. Let ω_1, ω_2 and ω_3 denote the corresponding frequency of atomic energy levels. Let us also denote ω_0 as the transition frequency between $|a\rangle$ and $|b\rangle$.

Denoting the transformation by T it is seen that if the initial state of the system at t=0 be $|\psi(0)\rangle$ then the solution of the Schrödinger equation at any later time t is

$$|\psi(t)\rangle = T(t)|\psi(0)\rangle \tag{4}$$

$$\bar{T}(t) = T^{-1}(t)$$

Using the expression of Hamiltonian given by (1), we obtain the wavefunction for the composite system of one atom interacting with the field. The method of T operator explained earlier has been used (Singh et. al., 2012).

$$\begin{aligned}
|\psi> = {} & [1 - it(\omega_0/2 + \omega_1 n_1 + \omega_2 n_2 + \omega_3 n_3) - t^2/2!\,[\omega_0/2 + \omega_1 n_1 + \omega_2 n_2 + \omega_3 n_3]^2 + g^2(n_1+1)(n_2+1)(n_3+1)] + it^3/3!\,[(\omega_0/2 + \omega_1 n_1 + \omega_2 n_2 + \omega_3 n_3)^3 + g^2(n_1+1)(n_2+1)(n_3+1)\{\omega_0/2 + 2\omega_1 n_1 + \omega_1(n_1+1) + 2\omega_2 n_2 + \omega_2(n_2+1) + 2\omega_3 n_3 + \omega_3(n_3+1)\} + \ldots\ldots\ldots]\,|a, n_1, n_2, n_3> \\
& + [-igt\sqrt{(n_1+1)}\sqrt{(n_2+1)}\sqrt{(n_3+1)} - gt^2/2!\,\sqrt{(n_1+1)}\sqrt{(n_2+1)}\sqrt{(n_3+1)}\,\{\omega_1(n_1+1) + \omega_1 n_1 + \omega_2(n_2+1) + \omega_2 n_2 + \omega_3(n_3+1) + \omega_3 n_3\}] \\
& + igt^3/3!\,\sqrt{(n_1+1)}\sqrt{(n_2+1)}\sqrt{(n_3+1)}\,\{g^2(n_1+1)(n_2+1)(n_3+1) + (\omega_0/2)^2 + \omega_1^2(n_1+1)^2 + \omega_1^2 n_1(n_1+1) + \omega_1^2 n_1^2 + \omega_2^2(n_2+1)^2 + \omega_2^2 n_2(n_2+1) + \omega_2^2 n_3^2 + \omega_3^2(n_3+1)^2 + \omega_3^2 n_3(n_3+1) + \omega_3^2 n_3^2 \\
& - (\omega_0/2)\omega_1 - (\omega_0/2)\omega_2 - (\omega_0/2)\omega_2 - (\omega_0/2)\omega_3 + 2\omega_1(n_1+1)\omega_2(n_2+1) + 2\omega_1 n_1\omega_2 n_2 + \omega_1(n_1+1)\omega_2 n_2 + \omega_1 n_1\omega_2(n_2+1) + 2\omega_2(n_2+1)\omega_3(n_3+1) + 2\omega_2 n_2\omega_3 n_3 + \omega_2(n_2+1)\omega_3 n_3 + \omega_2 n_2\omega_3(n_3+1) \\
& + 2\omega_1(n_1+1)\omega_3(n_3+1) + 2\omega_1 n_1\omega_3 n_3 + \omega_1(n_1+1)\omega_3 n_3 + \omega_1 n_1\omega_3(n_3+1)\} + \ldots\ldots\ldots]\,|b, (n_1+1), (n_2+1), (n_3+1)>
\end{aligned} \tag{5}$$

Atom-Atom Entanglement : We have used the method of concurrence put forward by Wooter to quantify the atom atom Entanglement (Wooters, 1998).

The state of the composite system has been found so that the density operatorof the two atoms and the field is given by

$$P\ atom\text{-}atom\text{-}field = |\psi(t)\rangle\langle\psi(t)| \quad (6)$$

Where 4 is given by eq. (5). The atom-atom density operator is given by

$$P\ atom\text{-}atom^{=Tr}\ fieldP\ atom\text{-}atom\text{-}field \quad (7)$$

Since the joint state of two atoms emanating from the cavity is not a pure state , the entanglement of the two-atom system can be quantified by the concurrence as proposed by Wootters. This has been widely used to study bipartite entanglement.

The concurrence of the system is given by

$$C(p) = \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\} \quad (8)$$

Where λ's are the four eigenvalues of the non-Hermitianmatrix

$$R=\rho(\sigma_y\otimes\sigma_y)\rho^*(\sigma_y\otimes\sigma_y) \quad (9)$$

arranged in a decreasing order, r is the (4 x 4) density matrix of the two atom system. Entanglement can be quantified by another function, called the entanglement of formation E_f (r), monotone of C. It can be defined as

$$E_f(\rho) = h\left[\frac{1+\sqrt{1-C^2(\rho)}}{2}\right] \quad (10)$$

where $h(x) = -x\log_2 x - (1-x)\log_2(1-x)$

The cavity field statistics could have considerable influence on the atom-atom entanglement.

Non degenerate case i.e. $\omega_1+\omega_2,+\omega_3 \approx \omega_o$

$$\hat{H} = 1/2\hbar\,\omega_o\hat{\sigma}_3 + \hbar\omega_1\hat{a}_1\dagger\hat{a}_1 + \hbar\,\omega_2\hat{a}_2\dagger\hat{a}_2 + \hbar\,\omega_3\hat{a}_3\dagger\hat{a}_3 + \hbar\, g$$

$$(\hat{a}_1\dagger\hat{a}_2\dagger\hat{a}_3 + \hat{\sigma}\_ + \hat{a}_1\hat{a}_2\hat{a}_3\hat{\sigma}_+) \text{-}(11)$$

Let the initial state be $a_1,n_1,n_2,n_3>$

Then, $\hat{T}(t) = \exp\{^- it[g(\hat{a}_1\dagger\hat{a}_2\dagger\hat{a}_3\dagger\sigma\text{-}+\hat{a}_1\hat{a}_2\hat{a}_3\sigma)]\}$

$\hat{T}(0) >= \hat{T}(t)$ (12)

$$|\psi_1(t)\rangle = T(t)|\psi(0)\rangle$$

- $\hat{M}+|a,n_1,n_2,n_3>=\sqrt{n_1}\sqrt{n_2}\sqrt{n_3}\,|\,0, n_1-1, n_2\text{-}1, n_3\text{-}1>=0$
- $\hat{M}+|a,n_1,n_2,n_3>=\sqrt{n_1+1}\sqrt{n_2+1}\sqrt{n_3+1}\,|\,b, n_1+1, n_2+1, n_3+1>$
- $\hat{M}+|b,n_1,n_2,n_3>=\sqrt{n_1}\sqrt{n_2}\sqrt{n_3}\,|\,a, n_1-1, n_2\text{-}1, n_3\text{-}1>$
- $\hat{M}-|b,n_1,n_2,n_3>=0$
- $\hat{M}+\hat{M}-|a,n_1,n_2,n_3>=(n_1+1)(n_2+1)(n_3+1)\,|\,a,n_1,n_2,n_3>$

  $\hat{M}+\hat{M}+|a,n_1,n_2,n_3>=0$
- $\hat{M}+^3|a,n_1,n_2,n_3>=0$
- $\hat{M}-^3|a,n_1,n_2,n_3>=0$
- $\hat{M}+\hat{M}-\hat{M}+|a,n_1,n_2,n_3>=$

  $\sqrt{n_1+1}\sqrt{n_2+1}\sqrt{n_3+1}$

  $(n_1+1)(n_2+1)(n_3+1)\ /\ b,n_1+1,n_2+1,n_3+1>$
- $\hat{M}-\hat{M}+\hat{M}-|a,n_1,n_2,n_3>=0$

Putting the value in equation (12)

$|\Psi_1(t)>=X\,|\,a_1,n_1,n_2,n_3>+Y\,|\,b_1,n_1+1,n_2+1,n_3+1>$

$X=\ \ldots=\cos(tg\sqrt{n_1+1}\sqrt{n_2+1}\sqrt{n_3+1})$

$Y=-i\sin(gt\sqrt{n_1+1}\sqrt{n_2+1}\sqrt{n_3+1})$

Now let us consider second atom entering the cavity field to be in higher state.

$|\Psi_2(t)>=T(t)>$

$\{X\,|\,a_2,a_1,n_1,n_2,n_3>+Y$

$|a_2,b_1,n_1+1,n_2+1,n_3+1>\}$

$|\Psi_2(t)\rangle = \alpha \,|\, a_2, a_1, n_1, n_2, n_3 \rangle$
$+ \beta \,|\, a_2, b_1, n_1+1, n_2+1, n_3+1\rangle + \gamma |b_2, a_1, n_1+1,$
$n_2+1, n_3+1\rangle + \delta |b_2, b_1, n_1+2, n_2+2, n_3+2\rangle$ (13)

Where,

$\alpha = \cos^2 tg\ \sqrt{n_1+1}\sqrt{n_2+1}\sqrt{n_3+1})$

$\beta = \cos(tg\ \sqrt{n_1+1}\sqrt{n_2+1}\sqrt{n_3+1})$

$[-i\sin\ (gt\ \sqrt{n_1+1}\sqrt{n_2+1}\sqrt{n_3+1})\ ]$

$\gamma = [\cos tg\ \sqrt{n_1+1}\sqrt{n_2+1}\sqrt{n_3+1})\ ]$

$[-i\sin tg\ \sqrt{n_1+2}\sqrt{n_2+2}\sqrt{n_3+2})\ ]$

$\delta = [-i\sin tg\ \sqrt{n_1+1}\sqrt{n_2+1}\sqrt{n_3+1})\ ]$

$[-i\sin tg\ \sqrt{n_1+2}\sqrt{n_2+2}\sqrt{n_3+2})\ ]$ (14)

according to equation (6), we have

$\rho = |\Psi(t)\rangle\langle\Psi(t)|$
$= |\alpha \,|\, a_2, a_1, n_1, n_2, n_3\rangle + \beta$
$|a_2, b_1, n_1+1, n_2+1, n_3+1\rangle + \gamma$
$|b_2, a_1, n_1+1, n_2+1, n_3+1\rangle + \delta$
$|b_2, b_1, n_1+2, n_2+2, n_3+2\rangle]$
$[\alpha^* \langle a_2, a_1, n_1 n_2 n_3| +$
$\beta^* \langle a_2, b_1, n_1+1, n_2+1, n_3+1| +$
$\gamma^* \langle b_2, a_1, n_1+1, n_2+1, n_3+1| +$
$\delta^* \langle b_2, b_1, n_1+2, n_2+2, n_3+2|$ (15)

The reduced mixed density state of the two atoms after tracing over the field.

$$\rho_{a-a} = \begin{bmatrix} \alpha^* \alpha & 0 & 0 & 0 \\ 0 & \beta^* \beta & \beta\gamma^* & 0 \\ 0 & \beta^* \gamma & \gamma\gamma^* & 0 \\ 0 & 0 & 0 & \delta^* \delta \end{bmatrix} \quad (16)$$

In the basis of $|a_1 a_2\rangle, |a_1 b_2\rangle, |b_1 a_2\rangle$ and $|b_1 b_2\rangle$ states. When the cavity field is in a photon number state $|n_1, n_2, n_3\rangle$ the four eigenvalues are given by

$\lambda_1 = 0$

$\lambda_2 = \lambda_3 = |\alpha|^2 |\delta|^2$

$\lambda_4 = \lambda_4 = |\beta|^2 |\gamma|^2$

$C = \max.\ \{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\}$;

$\lambda_1 > \lambda_2 > \lambda_3 > \lambda_4$

$= \max.\ \{0, \sin 2tg\ (n_1+1)^{1/2} (n_2+1)^{1/2} (n_3+1)^{1/2}$

$\times \sin tg\ \{(n_1+1)^{1/2} (n_2+1)^{1/2} (n_3+1)^{1/2} - (n_1+2)^{1/2}$

$(n_2+2)^{1/2} (n_3+2)^{1/2}\ ]\}$ For a vaccum state,

$n_1 = 0, n_2 = 0, n_3 = 0$ it reduces to

$C = \max\ \{0, \sin 2tg \sin tg\}$

Hence the concurrence of the two atom bipartite system is given by

Ef vs gt for three mode fock state with n=0

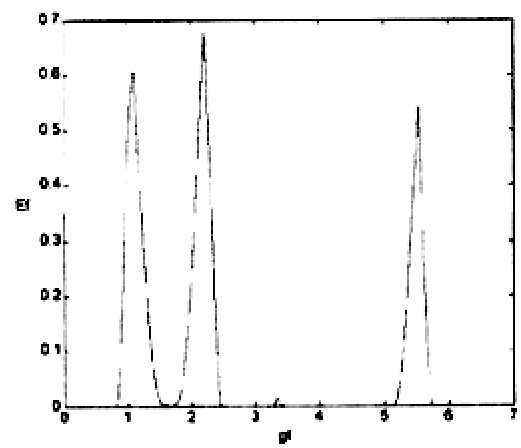

Fig. 4. Ef with $n_1, n_2, n_3 = 0$

Ef vs gt for three mode fock state with n=2

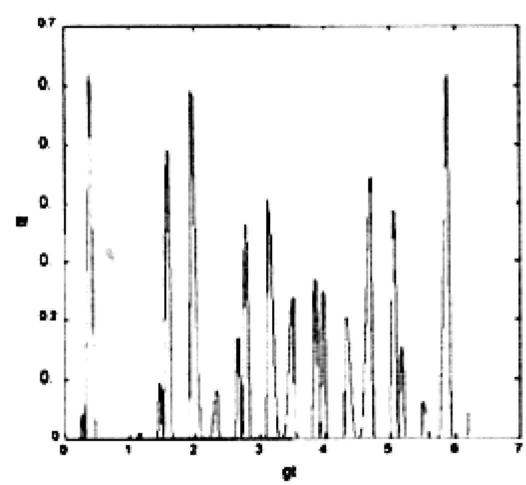

Fig. 5. Ef with $n_1, n_2, n_3 = 2$

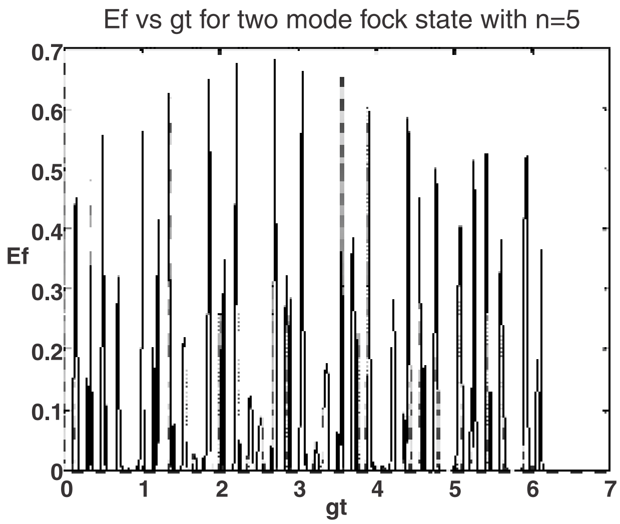

Fig. 6. Ef with n_1, n_2, n_3 = 5

In Fig 4,5,6, we plot the entanglement of formation which is a monotone of concurrence. There is a finite entanglement induced by the vacuum field. This is due to the fact that the first atom in excited state interacts with the vacuum cavity field and in this process disturbs the field. So the cavity field is no longer in the vacuum state when the second atom enters. This makes the atoms entangled. For n = 0, the argument of the sin function increases with increasing n. Hence during these oscillations, the concurrence get non-zero more frequently.

Conclusion :

From the analysis of atom-atom entanglement, it is found that the two atom show entanglement properties even when there is a vaccum field for all the three modes. It has also been observed in two atom three mode process, but the value of entanglement is more in this case both for n=0 and n=2. [fig.4 & 5]

We observe periodic entanglement and disentanglement ie.there is a series of collapse and revival of value of E_f. This property in fact follows from the collapse and revival in population of atomic levels (population inversion) taking place for two-level atoms in a cavity interacting with two modes of the radiation field. There can be seen a definite pattern in the plots if we plot for larger ranges of gt. The nature and properties of the radiation field have a crucial bearing on the magnitude of the generated atomic entanglement. The same method can be used for analysis when we consider coherent and thermal distribution of the cavity field.

The generalized models of standard JCM are paradigm to study the non-classical behaviour, particularly relating to correlation properties of atoms. The model can be extended to multiphoton process also, which can be the topic of further study.

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